

Waveguiding properties of optical vortex solitons

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We study the properties of linear and nonlinear waveguides induced by *optical vortex solitons* in both self-defocusing and self-focusing nonlinear media, for the case of a saturable nonlinear response. We demonstrate that the vortex-induced waveguides can guide both fundamental and first-order guided modes which together with the vortex may form, for larger amplitudes of the guide modes, different types of composite vortex-mode vector solitons. In the case of the self-focusing saturable medium, we demonstrate that a large-amplitude guided mode can stabilize the ring-like vortex structure which usually decays due to azimuthal modulational instability.

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I. INTRODUCTION

Vortices are fundamental localized objects which appear in many branches of physics, ranging from the physics of liquid crystals to the dynamics of superfluids and Bose-Einstein condensates [1]. Different types of vortices can be found and identified in optics; one of the simplest objects of this kind is a *phase singularity* in an optical wave front which is associated with a phase dislocation carried by a diffracting optical beam [2].

When such an optical vortex propagates in *self-defocusing nonlinear medium*, the vortex core with a phase dislocation becomes self-trapped, and the resulting *stationary singular beam* is known as an *optical vortex soliton* [3]. Such optical vortex solitons exist on a nonvanishing background wave, and they represent a two-dimensional generalization of the so-called *dark solitons* [4]. They can be generated in experiment as self-trapped singularities of broad beams by using a *holographic phase mask*, as was demonstrated for different types of nonlinear defocusing media [5, 6, 7]. As a matter of fact, this type of optical vortex solitons demonstrates many common features with the vortices observed in superfluids and Bose-Einstein condensates [8].

In *self-focusing nonlinear media*, optical vortices can exist as ring-like optical beams with zero intensity at the center carrying a phase singularity [9]. However, due to the self-focusing nature of nonlinearity such ring-like vortex beams become unstable, and they are known to decay into several fundamental optical solitons flying off the main ring [10]. This effect was observed experimentally in different nonlinear systems, including the saturable Kerr-like nonlinear media, biased photorefractive crystals, and quadratic nonlinear media in the self-focusing regime (see details and references in Ref. [4]).

Waveguides induced by optical vortices in both linear and nonlinear regimes are of a special interest because this type of waveguides is robust and can be made reconfigurable. Earlier numerical and experimental results [11, 12, 13] indicate that many of the vortex waveguiding properties should be similar to those of planar dark solitons [4]. Moreover, the vortex-induced wave-

guides can guide large-amplitude beams beyond the applicability limits of the linear guided-wave theory, and, together with the guided beam, they can form a special type of the *vortex-mode vector soliton* or its various generalizations [14, 15, 16, 17].

In this paper, we carry out a systematic analysis of the waveguiding properties of the vortex solitons and vortex-mode vector solitons in saturable nonlinear media, for both self-defocusing and self-focusing nonlinear media. We follow the earlier analysis of Ref. [13] and examine *two major regimes* of the vortex waveguiding. In the linear regime, the guided beam is weak, and the analysis of the vortex waveguiding properties is possible by approximate analytical methods, reducing the problem to a well-known analysis of the linear guided-wave theory (as an example, see Ref. [12]). The most interesting nonlinear regime corresponds to large intensities of the guided beam, and it gives rise to composite (or vector) solitons with a vortex component, that should be identified and analyzed numerically.

We also analyze the vortex waveguiding and vortex-mode vector solitons in a self-focusing nonlinear medium where, as is well known, the vortex beam becomes self-trapped and it also displays strong azimuthal modulational instability. However, we demonstrate that a mutual incoherent coupling between the vortex waveguide and a large-amplitude guided mode it guides can provide a strong stabilizing mechanism for stable two-component vortex solitons to exist in such media.

The paper is organized as follows. The next section (Sec. II) presents our model which takes into account both the incoherent coupling between the waveguide and the localized mode it guides, and the saturable nature of nonlinearity usually realized in experiment. The model is valid for both self-defocusing and self-focusing media. Section III is devoted to the study of the properties of optical vortex solitons and their guiding properties in self-defocusing media, whereas Sec. IV summarizes our results for the vortex waveguides and vector solitons in self-focusing nonlinear media. In particular, for the first time to our knowledge, we demonstrate that the vortex-mode vector solitons formed by the vortex waveguide together with a large-amplitude mode it guides can show

the *bistability* property, when two types of different composite soliton structures exist for the same value of the soliton power. We also discuss, by means of direct numerical simulations, the stability properties of the vortex-mode vector solitons in a self-focusing nonlinear medium. In particular, we reveal an effective method of suppression of the azimuthal instability through the stabilization effect that the guided mode exerts on the hosting vortex, the latter is known to be unstable by itself in focusing media. Finally, Sec. V concludes our paper.

II. MODEL

In order to study the guided modes of a vortex-induced waveguide in both self-focusing and self-defocusing media, we consider the interaction of two mutually incoherent optical beams propagating in a nonlinear saturable medium. The equations of motion for two beams can be presented in the dimensionless form as follows,

$$\begin{aligned} i\frac{\partial u}{\partial z} + \Delta_{\perp}u + \eta \frac{(|u|^2 + \mu|v|^2)u}{1 + \sigma(|u|^2 + |v|^2)} &= 0, \\ i\frac{\partial v}{\partial z} + \Delta_{\perp}v + \eta \frac{(|v|^2 + \mu|u|^2)v}{1 + \sigma(|u|^2 + |v|^2)} &= 0, \end{aligned} \quad (1)$$

where u and v are the dimensionless amplitudes of two fields, the parameter σ characterizes the nonlinearity saturation effect, and the incoherent mode interaction is described by the coupling parameter μ . The spatial coordinate z is the propagation direction of the beams, and Δ_{\perp} stands for the transversal part of the Laplacian operator in the cylindrical coordinates $r = (x^2 + y^2)^{1/2}$ and $\phi = \tan^{-1}(y/x)$. The sign parameter $\eta = \pm 1$ defines the type of the nonlinear medium under consideration, which is self-focusing, for $\eta = +1$, or self-defocusing, for $\eta = -1$, respectively.

The model (1) provides a straightforward generalization to a number of important cases studied earlier. In particular, the limit $\sigma \rightarrow 0$ corresponds to the Kerr medium discussed, for example, in Refs. [11, 13, 18], whereas the saturable case at $\mu = 1$ corresponds to the incoherent beam interaction in photorefractive nonlinear media where different types of composite vector solitons have been predicted theoretically and observed in experiment [14, 15, 16, 17]. In particular, some examples of the two-component vector solitons composed of a vortex component and the localized modes it guides were presented earlier by Haelterman and Sheppard [18] for the defocusing Kerr nonlinearity.

We look for stationary solutions of the system 1 in the form of a radially symmetric single-charged vortex beam in the field u of the form

$$u(r, \phi; z) = u(r)e^{i\phi}e^{i\gamma z}, \quad (2)$$

where γ is the propagation constant of the vortex mode, which if considered normalized takes the values $+1$ or -1 in the focusing and defocusing cases respectively, and

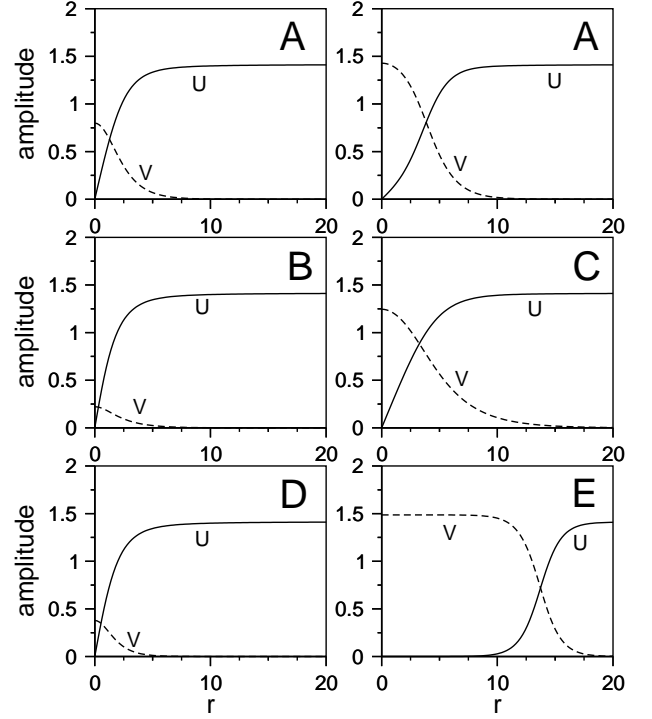


FIG. 1: Examples of localized solutions for the vortex waveguide (u) guiding a fundamental-mode beam (v). The labels correspond to the points marked in Fig. 2. Two examples labelled by the same letter A constitute two different types of localized solutions found in this domain (mode bistability). The parameters are: A ($\mu = 1.5$, $\beta = -1.1$), B ($\mu = 1.1$, $\beta = -0.85$), C ($\mu = 1.1$, $\beta = -1.0$), D ($\mu = 2.0$, $\beta = -1.3$), and E ($\mu = 2.0$, $\beta = -1.05$).

consequently we have $\gamma = \eta$. For $r \rightarrow \infty$, the amplitude $u(r)$ approaches a constant value, for the self-defocusing medium, or zero, for the self-focusing case, respectively. We assume that the vortex guides the second beam given by the expression

$$v(r, \phi; z) = v(r)e^{il\phi}e^{i\beta z}, \quad (3)$$

where β is the propagation constant of the guided beam. In the expressions (2), (3) the functions $u(r)$ and $v(r)$ are the radial envelopes of the interacting fields, and l is the angular momentum of the guided mode.

In this way, the following set of stationary z -independent equations are obtained:

$$\begin{aligned} -\eta u + \Delta_r u - \frac{1}{r^2}u + \eta \frac{(u^2 + \mu v^2)u}{1 + \sigma(u^2 + v^2)} &= 0, \\ -\beta v + \Delta_r v - \frac{l^2}{r^2}v + \eta \frac{(v^2 + \mu u^2)v}{1 + \sigma(u^2 + v^2)} &= 0, \end{aligned} \quad (4)$$

where $\Delta_r \equiv (1/r)d/dr(r d/dr)$. The model 4 describes different types of localized solutions for the vortex and the localized modes it guides, and it has been further analyzed by the numerical relaxation methods to classify its localized solutions for both defocusing and focusing cases, respectively.

In the defocusing case when $\eta = -1$, we have $u \rightarrow u_0$ and $v \rightarrow 0$ for $r \rightarrow \infty$. Using this asymptotic conditions, we obtain

$$\left(1 - \frac{u_0^2}{1 + \sigma u_0^2}\right) u_0 = 0, \quad (5)$$

and find the result for the vortex background amplitude u_0 in the form, $u_0 = (1 - \sigma)^{-1/2}$.

Using the second equation for v , and neglecting the terms which most quickly tend to zero when $r \rightarrow \infty$, we obtain:

$$-\beta v + \frac{d^2 v}{dr^2} - \frac{\mu u_0^2}{1 + \sigma u_0^2} v = 0. \quad (6)$$

In order to have the exponentially decaying function v (for $r \rightarrow \infty$), the following condition should be satisfied,

$$-\beta + \frac{\mu u_0^2}{1 + \sigma u_0^2} < 0. \quad (7)$$

that gives as the result

$$\beta + \mu > 0 \implies \beta > -\mu, \quad (8)$$

which defines the existence domain threshold as the line $-\beta = \mu$. This result is valid for both fundamental and first-order guided modes.

III. VORTEX GUIDED MODES

First, we consider the case of self-defocusing nonlinearity when $\eta = -1$. In this case, the vortex amplitude $u(r)$ approaches a finite-amplitude asymptotic value u_0 for $r \rightarrow \infty$, where u_0 can be found from the asymptotic analysis, $u_0 = (1 - \sigma)^{-1/2}$. With the help of numerical relaxation methods, we find various types of localized modes that describe a vortex and the localized guided mode with the amplitude $v(r)$ satisfying the condition $v(r) \rightarrow 0$ for $r \rightarrow \infty$. This condition requires that $\beta < 0$, since the field $v(r)$ is localized due to the presence of the vortex waveguide only, and it describes a guided mode.

Figure 1 shows several characteristic solutions of this type for particular values of μ and $\sigma = 0.5$, whereas the existence domain for the solutions with the fundamental guided mode (i.e. when $l = 0$) is shown in Fig. 2. For the values of the propagation constant β close to the cutoff ($\beta_c \approx -0.79$ at $\mu = 1$), the second field presents a low-amplitude guided mode that can be analyzed by means of the linear guided wave theory. However, for values of β close to the threshold, the amplitude of the guided mode becomes comparable to that of the vortex and its width grows, as shown for the case E in Fig. 1. In this latter case, the vortex is deformed as well, so the resulting structure describes a kind of a two-component vector soliton with the vortex-like mode.

In Fig. 3, we show the existence domain for the first vortex-like ($l = 1$) mode of the induced vortex waveguide.

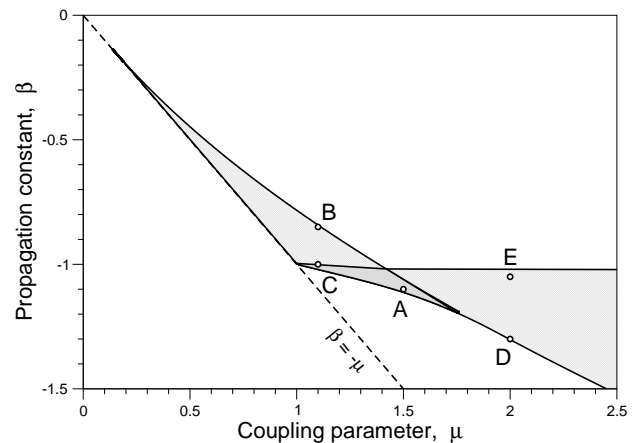


FIG. 2: Existence domain of the fundamental mode in the vortex-induced waveguides on the plane (μ, β) , for $\sigma = 0.5$. Marked points correspond to the solutions presented above in Fig. 1. A triangle region with the point A corresponds to the existence domain for bistable composite solitons.

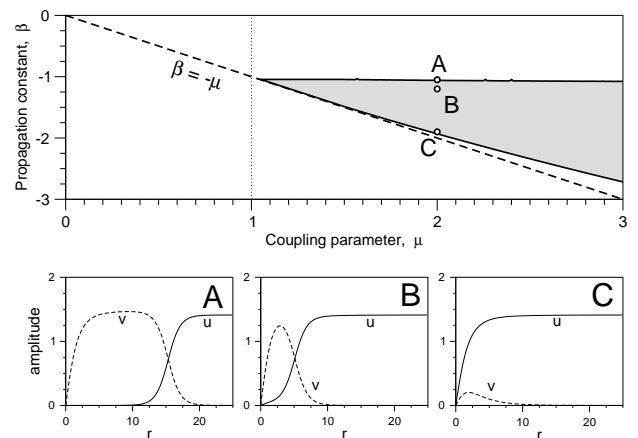


FIG. 3: Top: existence domain for the first-order mode ($l = 1$) guided by the vortex-induced waveguide, show on the plane (μ, β) . Below: examples of the two-component localized solutions for $\sigma = 0.5$, $\mu = 2$, and $\beta = -1.05$ (A), $\beta = -1.2$ (B), and $\beta = -1.9$ (C), respectively.

Similar to the case of the fundamental guided mode, the linear theory is applicable near the mode cut-off, as is shown for the mode C, where the mode grows and it deforms the vortex waveguide substantially, see the case A near the upper boundary of the existence domain in Fig. 3 (top).

IV. SELF-TRAPPED VORTEX BEAMS

A. Vortex-mode vector solitons

In the case of the self-focusing nonlinear medium, we put $\eta = +1$, and consider the self-trapped vortex beams with the asymptotic behavior $u(r) \rightarrow 0$ as $r \rightarrow \infty$. Be-

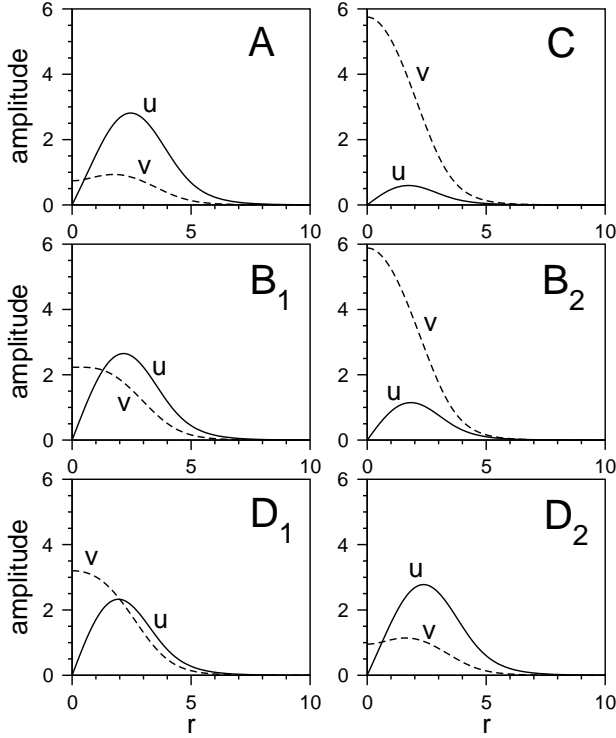


FIG. 4: Examples of solutions for the fundamental mode of a waveguide induced by a bright-like vortex. Labels correspond to the points marked in FIG. 4, using a subscript to name the two solutions found for the points B and C. Values: A ($\mu = 1.10, \beta = 1.38$), B ($\mu = 1.10, \beta = 1.44$), C ($\mu = 1.17, \beta = 1.38$), D ($\mu = 1.17, \beta = 1.47$).

sides that, it is required that $v(r) \rightarrow 0$ and $\beta > 0$, so that $v(r)$ describes a localized mode. At the origin ($r = 0$), the boundary condition for the vortex is $u = 0$ and, if we seek the solutions corresponding to the fundamental guided mode ($l = 0$), we have $v = v_0$ (or $dv/dr = 0$) for the field $v(r)$ at $r = 0$.

In Fig. 4, we show some examples of the two-component localized solutions which describes a fundamental (no nodes) beam guided by the self-trapped vortex in the form of a vortex-mode soliton. The existence domain for this kind of solutions is shown in Fig. 5 on the parameter plane (μ, β) , calculated numerically for $\sigma = 0.5$. Close to the cutoff, the guided mode has a small amplitude (see the cases A and D₂), and the vortex is only weakly distorted. However, when the propagation constant β is close to the upper threshold, the field $v(r)$ grows in the amplitude affecting strongly the vortex mode.

In order to describe the bistable vector solitons, in Fig. 6 we plot the bifurcation diagram of the two-component localized solutions, for the partial and total beam powers. The power of the composite vortex-mode solitons P_{total} originates at the bifurcation point O_1 where the mode v is small and it can be described by the linear theory. For larger value of β , the curve bends, and then it merges with the other partial power

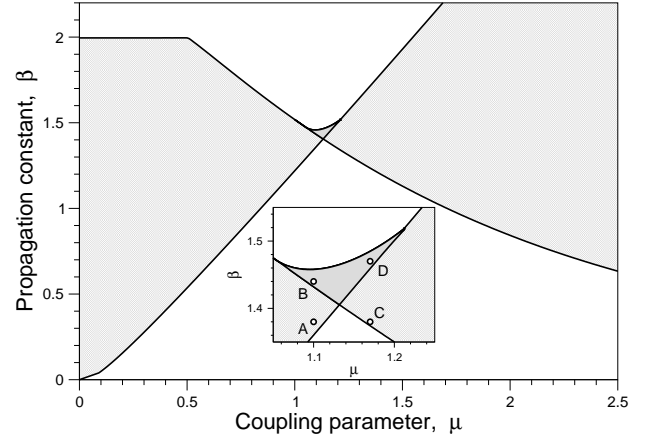


FIG. 5: Existence domain in the plane (μ, β) of the fundamental mode guided by the self-trapped vortex for $\sigma = 0.5$. Inset shows a zoomed region where two different types of solutions exist (bistability zone).

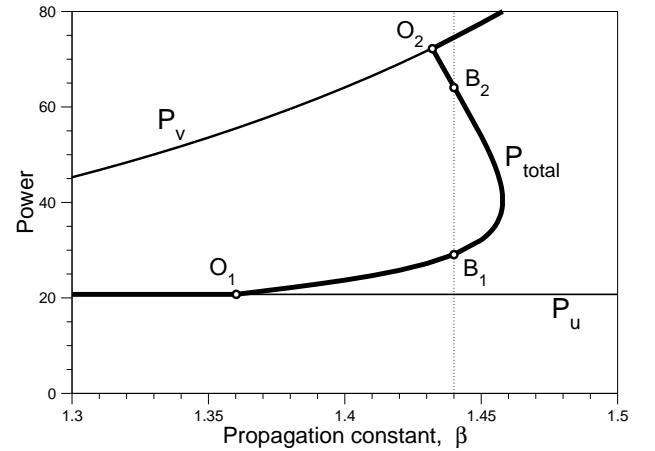


FIG. 6: Bifurcation diagram for the one- and two-component solitons composed by the bright vortex inducing a waveguide together with its fundamental mode for $\mu = 1.1$. Thin lines represent the powers P_u and P_v of each of the scalar solitons; the thick line is the total power P_{total} of the composite soliton. The points O_1 and O_2 are the bifurcation points where the vector soliton emerges, and the points B_1 and B_2 correspond to the examples of the bistable solutions for $\beta = 1.44$ shown in Fig. 4.

curve P_v at the bifurcation point O_2 (see Fig. 6). The bistable solutions B_1 and B_2 correspond to a single value of the propagation constant β in the bistability domain. Importantly, two solutions have different stability properties, and only one of the solutions is stable, as shown in Fig. 7.

B. Vortex stabilization

The incoherent interaction between the vortex and the localized mode it guides has the character of attraction,

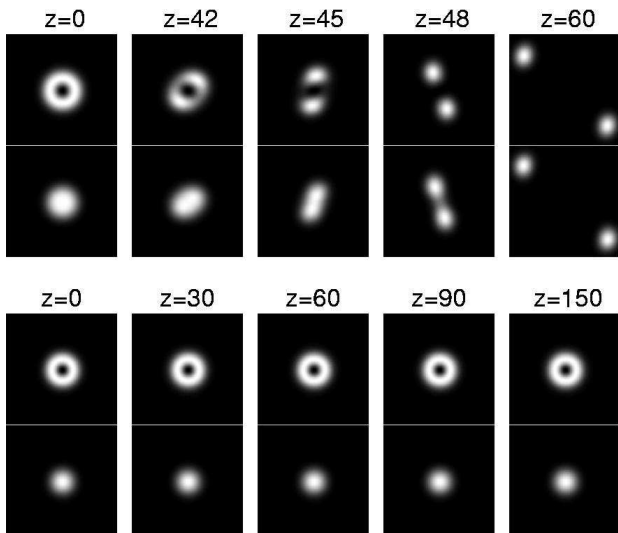


FIG. 7: Examples of the vortex evolution in the bistability domain. Shown are the field intensity profiles as gray-scale images at several propagation distances. Top row: the components of the vector soliton labelled as B_1 in Fig. 6. Bottom row: the components of the vector soliton labelled as B_2 in Fig. 6.

and it may provide an effective physical mechanism for stabilizing the vortex beam in a self-focusing nonlinear medium. Indeed, it is well-known that the vortex beam becomes unstable in a self-focusing nonlinear medium due to the effect of the azimuthal modulational instability. In this case, the vortex splits into the fundamental beams that fly off the main vortex ring [10]. On the other hand, the bright solitons are known to be stable in such media. We expect that a mutual attraction of the components in a two-component system would lead to a counter-balance of the vortex instability by the bright component if the amplitude of the latter is large enough.

To carry out this study, we consider a two-component composite structure consisting of a vortex beam together with the fundamental mode of the waveguide it guides, described by Eqs. 1 (at $\eta = 1$ and $\mu = 1.0$). To study the mode stability, we propagate the stationary soliton solutions numerically.

In Fig. 8, we compare the vortex stability for the scalar and two-component systems. In the top row, we show the propagation of a vortex alone in the scalar model; the vortex breaks into two solitons which fly away after some propagation distance. In the bottom row, we show the propagation of two coupled components (the vortex and bright mode it guides). Due to a strong coupling between the modes, the propagation of the vortex is stabilized and its decay can be delayed dramatically, as shown in Fig. 8(lower row), or become completely stable, similar to the case shown in Fig. 7(low row). We confirm this stabilization mechanism by performing a similar study for a Gaussian input beam of the bright component instead of the exact stationary state, as is easier to realize

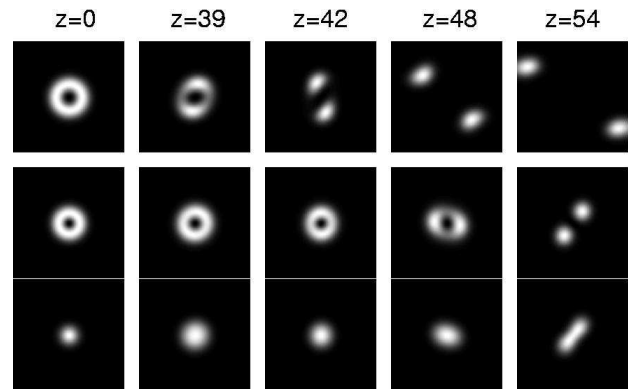


FIG. 8: Vortex stabilization in a two-component system. Shown are the field intensity profiles as gray-scale images at several propagation distances. Top row: propagation of a vortex soliton by itself ($v = 0$). Bottom row: both the vortex beam and fundamental mode (approximated by a Gaussian profile) propagate together. Parameters are: $\sigma = 0.5$, $\mu = 1$, and $\beta = 1.45$.

in experiment.

V. CONCLUSIONS

We have studied the properties of the vortex-induced optical waveguides in both self-focusing and self-defocusing saturable nonlinear media. We have calculated numerically the existence domains of the composite solitons created by an optical vortex and the localized mode it guides, and have analyzed the properties of the vortex waveguides in both linear and nonlinear regimes. In the case of self-defocusing nonlinear media, we have identified the regimes when the vortex soliton can guide both the fundamental and first-order guided modes creating an effective multi-mode waveguide. In the case of self-focusing nonlinear media, we have studied the effect of the strong coupling between the vortex waveguide and the guided mode on the vortex stability. In particular, we have described a novel mechanism of stabilizing the vortex azimuthal modulational instability by a large-amplitude guide mode, and we have revealed the existence of optical bistability for the vortex-mode composite solitons. We believe similar results can be obtained for vortices in other types of nonlinear models, and they can be useful for other fields such as the physics of the Bose-Einstein condensates of ultra-cold atoms.

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